A Parallel Algorithm for Broadcast Scheduling Problems in Packet Radio Networks

Nobuo Funabiki and Yoshiyasu Takefuji

***Abstract*— A parallel algorithm based on an artificial neural network model for broadcast scheduling problems in packet radio networks is presented. The algorithm requires $n \times m$ processing elements for an n-node–m-slot radio network problem. The algorithm is verified by simulating 13 different networks.**

I. Introduction

A PACKET radio network provides flexible data communication services among a large number of geographically distributed and mobile users (nodes). The network uses radio channels as the broadcast medium to interconnect nodes to offer flexibility for the change of network specifications. The time division multiple access (TDMA) technology has been used to share a radio channel among many users where a channel is divided into synchronized data packet transmission called "time slots." A time slot has a unit time to transmit one data packet per one node.

The packet radio network has the following constraints. First, the network has symmetric connectivity between nodes where any two connected or neighboring nodes can communicate with each other. Second, transmitted data packet may be received by all the neighboring nodes. Third, a node cannot send and receive a packet simultaneously. Fourth, a node cannot receive more than one packet simultaneously. If two or more nodes transmit packets to a node simultaneously, the node cannot receive any of them successfully. Network connectivity is described in an $n \times n$ symmetric binary matrix for an n-node network where an element N_{ij} is 1 if node i is a neighbor of node j; 0 otherwise. Fig. 1 shows a seven-node network where the edges represent the connectivity and the network matrix.

The broadcast scheduling problem in packet radio networks has been studied extensively. Baker *et al.* [1], [2] first formulated the problem and proposed some ad hoc algorithms. The problem of finding a TDMA cycle where all nodes can broadcast packets has been studied most intensively. Even *et al.* [3] proved that the TDMA-cycle minimization problem is NP-complete. In 1989, Cidon and Sidi [7] proposed two distributed algorithms: the round-robin algorithm where

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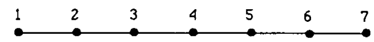

(a)

$$\begin{bmatrix} 0&1&0&0&0&0&0 \\ 1&0&1&0&0&0&0 \\ 0&1&0&1&0&0&0 \\ 0&0&1&0&1&0&0 \\ 0&0&0&1&0&1&0 \\ 0&0&0&0&1&0&1 \\ 0&0&0&0&0&1&0 \end{bmatrix}$$

(b)

Fig. 1. Seven-node packet radio network: (a) seven-node packet radio net work; (b) corresponding network matrix.

the node priorities are cycled, and the wait-for-neighbors algorithm where assigned nodes are eliminated from the following procedure before all neighboring nodes are assigned. In 1990, Ephremides and Truong [8] proposed centralized and distributed algorithms to find a maximal schedules where each node is assigned at each time slot diagonally; nodes with higher priorities are assigned in the remaining time slots.

This paper proposes a centralized parallel algorithm to find a conflict-free time slot assignment in a TDMA cycle. We assume that each node broadcasts one packet in a TDMA cycle whose length is given. The algorithm is based on the artificial neural network, which is composed of a large number of simple processing elements (neurons). The neural network for combinatorial optimization problems was first introduced by Hopfield and Tank [10]. Takefuji and his collaborators have modified the basic Hopfield–Tank model and applied it to several NP-complete and optimization problems [11]–[15]. In 1989 Tassiulas *et al.* [9] first proposed a Hopfield neural network for broadcast scheduling problems in radio networks. Tassiulas's model uses the decay term in the motion equation where it disturbs the system convergence on some conditions and provides poor solution quality.

Among several proposed neuron models, the hysteresis McCulloch–Pitts model [14] is used in this paper because it is empirically shown not only to decrease the number of infeasible solutions as compared to the sigmoid model [10] but also suppresses the oscillatory behavior of the McCulloch–Pitts model [16]. The output V_{ij} of the ijth processing element based on the hysteresis McCulloch–Pitts model follows:

$$\begin{aligned} V_{ij} &= 1, \quad \text{if } U_{ij} > \text{UTP (upper trip point)} \\ &= 0, \quad \text{if } U_{ij} < \text{LTP (lower trip point)} \\ &= \text{unchanged otherwise,} \end{aligned} \tag{1}$$

where U_{ij} and V_{ij} are the input and the output of the ijth processing element and UTP is always larger than LTP. The change of U_{ij} is given by the partial derivatives of the computational energy $E(V_{11}, \cdots, V_{nm})$, which is given by considering the necessary and sufficient constraints in the

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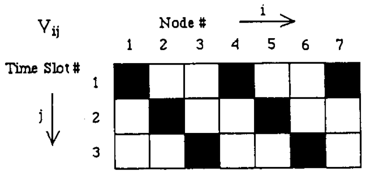

Fig. 2. System representation for network problem in Fig. 1.

problem. Note that n and m are the number of nodes and the number of time slots in a TDMA cycle, respectively. The motion equation is given by

$$\frac{dU_{ij}}{dt} = -\frac{\partial E(V_{11}, \cdots, V_{nm})}{\partial V_{ij}}. \tag{2}$$

The motion equation is proven to force the state of the system to converge to a local minimum [13].

II. System Representation

Fig. 2 shows the system representation for the problem in Fig. 1 where a seven-node network and three time slots are given. A total 21 ($= 7 \times 3$) processing elements are required in this problem, where three processing elements are used for each node. Generally $n \times m$ processing elements are required for an n-node$-m$-time-slot problem. The output of the ijth processing element represents the assignment of node i on time slot j. The nonzero output ($V_{ij} = 1$) means that node i is assigned on time slot j. Fig. 2 also shows one of the solutions, where the black square and the white square indicate the nonzero output and the zero output of the processing element, respectively.

A node cannot send and receive a packet simultaneously. In other words, if node i is assigned on time slot j, no neighboring nodes must be assigned on time slot j. The packet-send/receive simultaneous constraint is introduced by

$$\sum_{\substack{p=1\\p\neq i}}^{n} N_{ip} V_{pj}. \tag{3}$$

A node cannot receive more than one packet simultaneously. In other words, if node i is assigned on time slot j, no neighboring nodes of node i except node i itself should be assigned on time slot j. The one-packet-receive constraint is introduced by

$$\sum_{\substack{p=1\\p\neq i}}^{n} N_{ip} \sum_{\substack{q=1\\q\neq i\\q\neq p}}^{n} N_{pq} V_{qj}. \tag{4}$$

Therefore the motion equation of the ijth processing element for the n-node network on m time slots is given by

$$\frac{dU_{ij}}{dt} = -A\left(\sum_{p=1}^{m} V_{ip} - 1\right)$$

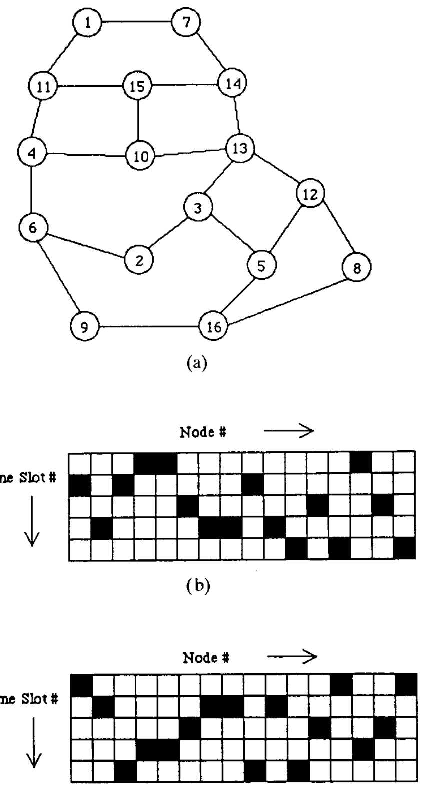

Fig. 3. Sixteen-node packet radio network problem: (a) sixteen-node packet radio network; (b) solution 1; (c) solution 2.

$$-B\left(\sum_{\substack{p=1\\p\neq i}}^{n} N_{ip} V_{pj} + \sum_{\substack{p=1\\p\neq i}}^{n} N_{ip} \sum_{\substack{q=1\\q\neq i\\q\neq p}}^{n} N_{pq} V_{qj}\right). \tag{5}$$

The first term (A) encourages one and only one output among m processing elements to be nonzero where node i is assigned. The second term (B) represents the constraints introduced in (3) and (4). The energy function E is given by (2) and (5) as

$$E = \frac{A}{2}\sum_{i=1}^{n}\left(\sum_{p=1}^{m} V_{ip} - 1\right)^2 + B\sum_{i=1}^{n}\sum_{j=1}^{m} V_{ij} \cdot \left(\sum_{\substack{p=1\\p\neq i}}^{n} N_{ip} V_{pj} + \sum_{\substack{p=1\\p\neq i}}^{n} N_{ip} \sum_{\substack{q=1\\q\neq i\\q\neq p}}^{n} N_{pq} V_{qj}\right). \tag{6}$$

The following three heuristics are used to increase the likelihood of convergence to the global minimum convergence [12]–[15].

TABLE I
SPECIFICATIONS OF SIMULATED PACKET RADIO NETWORKS AND SIMULATION RESULTS

Problem No.	No. of Nodes, n	No. of Edges	No. of Time Slots, m	Average Number of Iterations	Convergence frequency, %
1	8	9	5	47.5	100
2	14	23	6	165.2	35
3	16	23	5	209.8	26
4	36	62	9	69.4	96
5	64	122	9	145.9	66
6	100	114	6	82.6	97
7	100	189	9	135.5	67
8	100	282	12	179.0	65
9	144	279	10	137.9	81
10	196	370	10	124.1	89
11	256	499	10	135.0	85
12	324	643	10	169.1	64
13	400	805	10	242.8	37

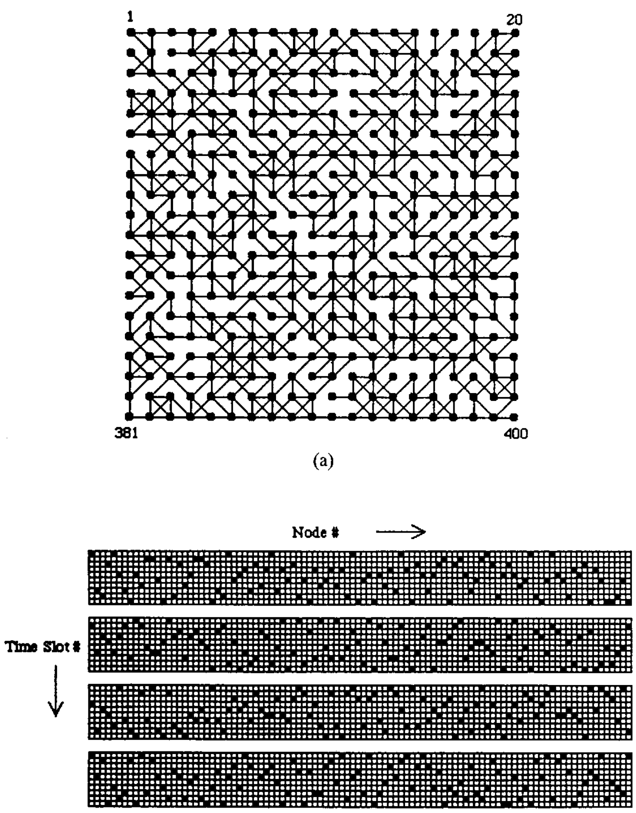

Fig. 4. 400-node packet radio network problem: (a) 400-node packet radio network; (b) solution.

1) The hill-climbing function:

$$h\left(\sum_{p=1}^{m} V_{ip}(t)\right), \quad \text{where } h(x) \text{ is 5 if } x = 0; \text{ 0 otherwise.} \tag{7}$$

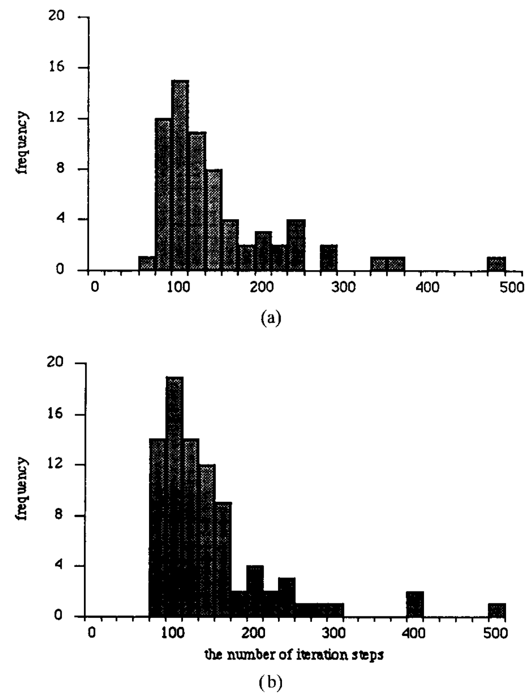

Fig. 5. Relationship between number of iteration steps to converge to solutions and the frequency: (a) 100-node network problem (problem 7); (b) 256-node network problem (problem 11).

2) The omega function—multiplying by the following function:

$$\begin{aligned} \Omega_{ij}(t) &= V_{ij} \ (t \bmod 10) < 5 \\ &= 1, \quad \text{otherwise.} \end{aligned} \tag{8}$$

3) The input saturation:

$$\begin{aligned} U_{ij} &= 20, \quad \text{if } U_{ij} > 20 \\ U_{ij} &= -20 \quad \text{if } U_{ij} < -20. \end{aligned} \tag{9}$$

The hill-climbing function helps the system to escape from a local minimum. The second and third heuristics make a local minimum shallower.

III. Simulation Results and Disscussion

The simulator based on the procedure in [11]–[15] with the proposed motion equation and three heuristics has been developed on Macintosh SE/30 and IIfx. the coefficients $A = B = 1$ were always used to show the robustness of our algorithm in simulation. The initial values of $U_{ij}(t)$ were set to uniform random numbers between 0 and -20. Table I shows the specifications of 13 simulated radio networks. Problem 1 is given from [9], problem 2 from [8], and problem 3 from [7], where the simulator found the minimum TDMA cycle. Figs. 3 and 4 show the network topology and solutions in two problems, respectively. Our simulator found several solutions in the same problem from different initial values of $U_{ij}(t)$. Table I also shows the average number of iteration steps and the convergence frequency to solutions where 100 simulation runs were performed for each problem. Fig. 5 shows the relationship between the number of iteration steps to solutions and the frequency in two problems. The simulation results empirically show that with $n \times m$ processors our algorithm finds a time slot assignment in an n-node–m-time-slot radio network in nearly constant time.

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