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Discussion

Bayesian analysis with inaccurate rapid antigen testing for detecting someone is positive

Yoshiyasu Takefuji

Faculty of Data Science, Musashino University, 3-3-3 Ariake Koto-ku, Tokyo 135-8181, Japan

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ABSTRACT

This paper examines how testing accuracy plays a key role in detecting someone is positive. Bayesian-theorem exploits how an inaccurate rapid antigen test with a high true-positive rate and a low false-positive rate affects the detection of the probability that someone is positive. The probability is calculated with three determinants in actual dental practices: Infection rate, true positive rate, and false positive rate. The result suggests that in a high infection rate, the false positive rate of rapid antigen tests plays a key role in detecting positive individuals. In 6.5% infection rate, 0.991 true positive rate, and 0.0005 false positive rate, the probability is 0.992 while it with 0.05 false positive rate is reduced to 0.579. The proposed Bayesian analysis can be used for future analysis with imprecise tests in other applications.

1. Introduction

The goal of this paper is to introduce the Bayesian analysis method on how inaccurate rapid antigen testing with confusion matrices can identify infected individuals in dental practices. The confusion matrix is a matrix used to determine the performance of the classification model for a given set of test data. In other words, the confusion matrix is a table with two rows and two columns that reports the number of true positives, false negatives, false positives, and true negatives. This paper examines how testing accuracy with the confusion matrix using real data plays a key role in detecting positive or infected individuals in dental practices.

The probability to visit asymptomatic COVID-19-infected patients in dentists in Italy was estimated: 1.2% in 2020 [1]. The probabilities of developing COVID-19 per worked hour per person excluding and including this uncertain situation were calculated from 0.0% to 0.9%. Therefore, the study concluded that relatively simple infection control procedures were enough to control occupational COVID-19 risk during the outbreak.

There are three infection types: pre-symptomatic, asymptomatic and symptomatic. All three types can spread COVID-19. Asymptomatic means there are no symptoms. Symptomatic can mean showing symptoms. Therefore, it is essential to detect asymptomatic and pre-symptomatic individuals at an early stage.

According to global percentage of asymptomatic SARS-CoV-2 infections among the tested population and individuals, the pooled percentage of asymptomatic infections was 0.25% [2].

However, Wachter stated that 6.5% of asymptomatic people getting tested at UCSF are positive for COVID-19, a slight uptick from 5% to 6% over the past two months from June 2022 [3]. If 6.5% of an asymptomatic population has Covid, in a crowd of 50 people, there is a 96% probability that someone there is positive.

E-mail address: takefuji@keio.jp.

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2. Methods and results

Bayes' theorem is useful for calculating the probability that someone is positive. Bayes' theorem is described by conditional probability. We will examine and calculate the probability of detecting that someone is positive with high infection rate.

Assume D and R :

D = event that individual has disease

R = event of a positive result

Assume that 6.5% of an infected population has COVID-19. The probability of individual with disease is given:

$$P(D{=}y) = 0.065$$

Therefore, non-infected population probability is: $P(D{=}n) = 1 - P(D{=}y) = 0.935$

Assume the true positive rate of testing is 0.991 according to Routsias [4] and the false positive rate is 0.0005 according to Gans [5]. The true positive means that individual with disease is detected as positive. The false positive means that individual with no disease is detected as positive.

$$P(R = y|D = y) = 0.991 \text{(true positive rate)}$$

$$P(R = y|D = n) = 0.0005 \text{(false positive rate)}$$

Therefore, the probability of positive result is given by:

$$\begin{aligned} P(R = y) &= P(R = y|D = y)P(D = y) + P(R = y|D = n)P(D = n) \\ &= 0.991 * 0.065 + 0.0005 * 0.935 = 0.0648825 \end{aligned}$$

The probability of detecting that someone is positive is given by:

$$\begin{aligned} P(D = y|R = y) &= P(R = y|D = y)P(D = y)/P(R = y) \\ &= 0.991 * 0.065/0.0648825 = 0.992 \end{aligned}$$

As the result, the probability of detecting that someone is positive is 0.992.

In other words, the true positive rate should be as high as possible and the false positive rate as low as possible to obtain the maximum positive detection.

If false positive rate is 0.05 instead of 0.0005 and 6.5% of the same infection population,

$$\begin{aligned} P(D = y|R = y) &= (0.991 * 0.065)/(0.991 * 0.065 + 0.05 * 0.935) \\ &= 0.579 \end{aligned}$$

The probability of detecting someone is positive is dramatically reduced to 0.579.

This calculation result indicates that the false positive rate and the true positive rate play a key role in detecting someone is positive. The higher the true positive rate and the lower the false positive rate are essential in testing for detecting positive. The better the detection, the stronger the COVID-19 mitigation. In other words, with a high infection rate, for the more protection, rapid antigen tests of the higher true positive rate and the lower false positive are needed in dental practices.

The probability of detecting someone is positive: P(x,y) can be formulated by the following equation where x and y is the false positive rate and the true positive rate respectively. Assume that the probability of individual with disease is 0.065 and non-infected population probability is 0.935.

$$P(x, y) = (y * 0.065)/(y * 0.065 + x * 0.935)$$

If y=0.991 and y=0.5, the graph of P(x,0.991) and P(x,0.5) will be drawn as shown in Fig. 1. Fig. 1 shows that the false positive rate significantly affects the probability of detecting someone is positive. In other words, the false positive rate should be as small as possible for higher probability of detecting someone is positive.

3. Conclusion

The proposed Bayesian analysis method showed that in a high infection rate, the more protection against COVID-19 is needed in dental practices. The better the rapid antigen testing, the higher the positive detection. The higher the positive detection, the better the COVID-19 mitigation. In a high infection rate, for the more protection, more rapid antigen tests of the higher true positive rate and the lower false positive are needed for better detection. Rapid antigen tests with the higher true positive rate and the lower false positive play a key role in detecting someone is positive. As a future work, due to the false positive/true positive relationship, it seems critical to be able to select a testing method for a given asymptomatic infection rate for future infections.

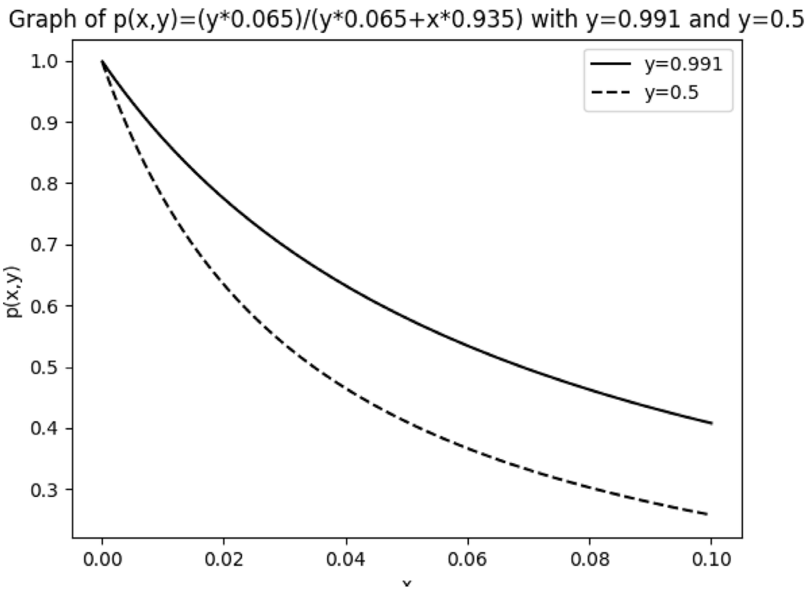

Fig. 1. The probability of detecting someone is positive: P(x,y) with y=0.991 and y=0.5.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

References

[1] M. Nardone, A. Cordone, S. Petti, Occupational COVID-19 risk to dental staff working in a public dental unit in the outbreak epicenter, Oral Dis. 28 (Suppl 1) (2022) 878–890, http://dx.doi.org/10.1111/odi.13632.

[2] Q. Ma, J. Liu, Q. Liu, et al., Global percentage of asymptomatic SARS-CoV-2 infections among the tested population and individuals with confirmed COVID-19 diagnosis: A systematic review and meta-analysis, JAMA Netw. Open. 4 (12) (2021) e2137257, http://dx.doi.org/10.1001/jamanetworkopen.2021.37257.

[3] A. Graff, UCSF's Dr. Bob Wachter says COVID subvariant BA.5 'destined to be our dominant virus', 2022, https://www.unitedag.org/news/ucsfs-dr-bob-wachter-says-covid-subvariant-ba.5-destined-to-be-our-dominant-virus/.

[4] J.G. Routsias, M. Mavrouli, P. Tsoplou, et al., Diagnostic performance of rapid antigen tests (RATs) for SARS-CoV-2 and their efficacy in monitoring the infectiousness of COVID-19 patients, Sci. Rep. 11 (2021) 22863, http://dx.doi.org/10.1038/s41598-021-02197-z.

[5] J.S. Gans, A. Goldfarb, A.K. Agrawal, S. Sennik, J. Stein, L. Rosella, False-positive results in rapid antigen tests for SARS-CoV-2, JAMA 327 (5) (2022) 485–486, http://dx.doi.org/10.1001/jama.2021.24355.